



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$BM: l\alpha + n\gamma = 0, \quad BQ: p\alpha + r\gamma = 0,$$

$$CN: l\alpha + m\beta = 0, \quad CR: p\alpha + q\beta = 0,$$

$$RM: lp\alpha + lq\beta + np\gamma = 0,$$

$$QN: lp\alpha + mp\beta + lr\gamma = 0.$$

At the intersections of BM and CN , of BQ and CR , and of RM and QN , the coördinate ratios are respectively:

$$\alpha_1 : \beta_1 : \gamma_1 :: -mn : nl : lm,$$

$$\alpha_2 : \beta_2 : \gamma_2 :: -qr : rp : pq,$$

$$\alpha_3 : \beta_3 : \gamma_3 :: l^2rq - p^2mn : p^2nl - l^2rp : p^2lm - l^2pq.$$

The vanishing of the determinant of these three sets of coördinates proves the proposition.

Also solved by NATHAN ALTSHILLER, H. C. FEEMSTER, and F. M. MORGAN.

444. Proposed by S. A. COREY, Hiteman, Iowa.

Let $ABCDE$ be a pentagon, plane or gauche, with sides AB, BC, CD, DE , and EA . Bisect BC and DE in H and K respectively. Extend AB from B to B' , and AE from E to E' . On AB' take sects AP and AV , and on AE' take sects AL and AT . Draw AD, AC, AH, AK , and DT . Let a, b, c , and d equal $AL/AE, AT/AE, AV/AB$, and AP/AB , respectively. Extend (or contract) AC from C to W , and AD from D to S , making $AW = a \times AC$ and $AS = d \times AD$. Draw LM and PN parallel to, and of the same currency as, AD and AC respectively, and of lengths $c \times AD$ and $b \times AC$, respectively. Draw AM, AN, ST , and WV . Draw DQ and VX parallel to, and of the same currency as, CB and TS , respectively. We are to prove that $2(ad + bc)(AK \times AH \times \cos KAH + KE \times HC \times \cos QDK) = AM \times AN \times \cos MAN + TS \times VW \times \cos WVX$.

SOLUTION BY THE PROPOSER.

Let KE, HB, AH and AK , in the proposed figure, be represented by the vectors x, y, z , and w , respectively. Then by vector addition, $w + x = AE, w - x = AD, z + y = AB, z - y = AC$; by construction, $\angle ST \cdot WV = \angle WVX, \angle KE \cdot HB = \angle QDK, (w + x)a = AL, (w - x)c = LM, (z + y)d = AP, (z - y)b = PN, (w + x)b = AT, (w - x)d = AS, (z + y)z = AV$, and $(z - y)a = AW$; also by vector addition, $(w + x)a + (w - x)c = AM, (z + y)d + (z - y)b = AN, (w + x)b - (w - x)d = ST$, and $(z + y)c - (z - y)a = WV$.

Consider now the algebraic identity,

$$(w + x)a + (w - x)c[(z + y)d + (z - y)b] \\ + [(w + x)b - (w - x)d][(z + y)c - (z - y)a] = 2(ad + bc)(wz + xy) \quad (1)$$

and note that when fully expanded each term is of the second degree in x, y, z , and w . Also note that the identity may be written

$$AM \times AN + ST \times WV = 2(ad + bc)(AK \times AH + KE \times HB) \quad (2)$$

if we substitute vectors as above indicated.

Inasmuch as vector multiplication is commutative if no term of the product is of a degree higher than the second in the vectors employed, and if the scalar part only of the resulting product be considered, we may assume that the algebraic identity (1) has a geometric interpretation which may be derived from (2) by considering the scalar part only of the vector products indicated in (2). The scalar part of the product of two vectors may be taken as the positive product of the lengths of the vectors into the cosine of their included angle. Placing this interpretation on the scalar part of the products indicated in (2) the equation of the problem is at once obtained, and the truth of the theorem established.

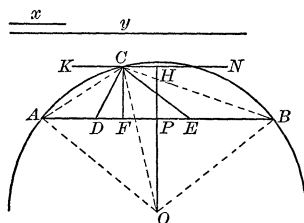
As no assumption has been made restricting any of the lines to any one plane the pentagon may, of course, be either plane or gauche. To help form a mental picture of a gauche pentagon, let $ABCD$ be a tetrahedron, with edges AB , BC , CD , DA , AC , and BD , and let E be a point within or without such tetrahedron. Then if E be connected with A and D by right lines the figure $ABCDE$ will be a gauche pentagon.

445. Proposed by CLIFFORD N. MILLS, South Dakota State College.

Given the perimeter of a right triangle and the perpendicular falling from the right angle on the hypotenuse, to determine the sides of the triangle.

SOLUTION BY G. I. HOPKINS, Manchester (N. H.) High School.

Let x be the altitude and y the perimeter. Draw $AB = y$, and OH its \perp bisector. Make $PH = x$, and draw KN through H \perp to HO . Make $PO = AP$. With O as center and radius OA describe the circle ACB . Draw the chords CA and CB , and the radii OA , OB , and OC . Make $\angle ACD = \angle CAD$, and $\angle BCE = \angle CBE$. $\therefore DCE$ is the \triangle required. For,



$$\angle AOP = 45^\circ = \angle POB. \therefore \angle AOB = 90^\circ.$$

$$\angle OAC = \angle OCA \text{ and } \angle CAD = \angle ACD. \therefore \angle OCD = \angle OAD = 45^\circ.$$

In like manner $\angle OCE = 45^\circ$; $\therefore \angle DCE = 90^\circ$, $AD = DC$, and $EB = CE$; \therefore the perimeter of the $\triangle DCE = y$, CF is \perp to AB , $\therefore CF = HP = x$.

Note. The figure is not accurate as OP is not made equal to AP .

Also solved by B. J. BROWN, A. H. HOLMES, C. N. SCHMALL, and NATHAN ALTSHILLER.

CALCULUS.

356. Proposed by F. B. FINKEL, Drury College.

A steel girder l ft. long and w ft. wide is moved along a passageway a ft. wide and into a corridor at right angles to the passageway. How wide must the corridor be to admit the girder?